International Baccalaureate: Extended Essay

Investigation of the effects of slipping on the motion of a cylinder rolling down an inclined plane

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Physics

Abstract:

This essay studies cylinder motion on an inclined plane in an attempt to answer the question: "how does the phenomenon of slipping affect the motion of a cylinder on an inclined plane?" The investigation attempts to answer this question on two levels: first, by comparing the linear and angular velocity of the cylinder at a point on the plane with and without slipping. Second, by investigating how the ratio of linear kinetic energy (K.E.) to rotational K.E. varies when there is slipping or not. A smooth wooden board and a plastic cylinder were used throughout the investigation. The cylinder was released from a point on the plane and a digital motion detector measured the velocity of the cylinder at another point down the plane for a range of angles. The velocity values were then compared to theoretical one, showing that the experimental velocity increased substantially relative to the theoretical one after about 35-40°. Regarding angular velocities, data showed that past the 35-40° range, the experimental values decreased appreciably compared the theoretical ones. Finally, the expected energy ratio of 2:1 linear K.E. to rotational K.E. from theory broke down when slipping occurred, and increased after the typical 35-40° range. Overall, this investigation has shown evidence that during slipping a freely rolling cylinder translates more and rotates less about its center of gravity than without slipping. This investigation has been limited to one type of both cylinder and plane surface. The study is merely an attempt at understanding better the phenomenon of slipping in cylinder motion and so no quantitative generalization has yet been made. A continuation of this investigation could possibly lead to the study of slipping of wheels in cart models as a way to apply the conclusions found in this investigation.

Word Count: 290.

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Introduction and Presentation:
1. Scope of work:

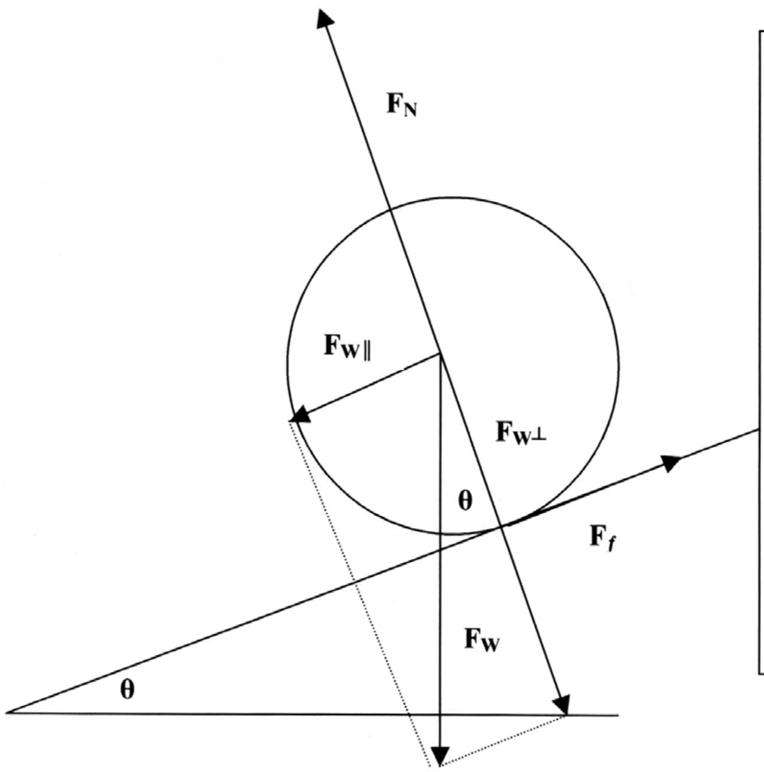
The rolling motion of a cylinder on an inclined plane is one that seems relatively simple yet one that becomes increasingly complex when slipping occurs. Because most high-school physics textbooks barely, if at all, cover the condition of slipping in simple rolling motion, it came to mind that investigating such phenomenon might prove interesting.

This essay is an attempt to study the phenomenon of slipping in the motion of a cylinder and to investigate its consequences. Further attention is given to the simple experiment of a cylinder rolling down an inclined plane to identify cases of slipping as the angle of elevation is raised and, if possible, to use of theory to describe this phenomenon. The first objective of this piece of work is to investigate how varying the angle of an inclined plane impacts the type of motion that the cylinder has. It includes taking a look at the linear and angular velocities when the cylinder slips compared to when the cylinder doesn't slip and just purely rolls. The second objective of this essay is to determine how the ratio of energy stored in linear kinetic energy (K.E.) to that in rotational K.E. changes when there is slipping or not. An evaluation of the reliability of the measurements and claims assesses the validity of the conclusions made on this subject.

2. Background Information and Literature:

Let us consider a cylinder rolling freely down an inclined plane as represented in diagram 1 below:

Diagram 1: Theoretical cylinder on an inclined plane:



With:

F_f: Friction force acting on the cylinder due to surface friction with board

F_N: Normal force from the board acting on the cylinder perpendicular to the plane.

θ: Angle of elevation of the inclined plane

F_W: Weight of cylinder (mg)

$F_{W\parallel}$: Component of the weight of the cylinder parallel to the plane (mgsin θ)

$F_{W\perp}$: Component of the weight of the cylinder perpendicular to the plane (mgcos θ)

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The friction force is the source of the cylinder's rolling motion because it creates a torque about its center of mass, which makes the cylinder roll.

Because the friction force is proportional to the normal force, we have: $F_f = \mu_r F_N = \mu_r mg\cos\theta$ (Hecht 145), with μ_r being the coefficient of rolling friction. If we take a closer look, as θ increases cos θ decreases and the friction force will be gradually smaller. We predict there is a point when the friction force will too small to provide a large enough torque to make the cylinder roll as fast as expected by theory, which will lead to slipping. Slipping occurs in that case when the instantaneous velocity at the point of contact between a rolling body and the medium supporting it is not zero (Hecht 306). That is, it rolls and 'slides' in its direction of motion at the same time.

Concerning our second objective, background information states that both the linear velocity (v) and the angular velocity (ω) are related together by the equation: $v = r\omega$ for normal rolling motion (Hecht 302). Using equations for gravitational potential energy ($E_{potential}$ or E_p), linear kinetic energy ($E_{kinetic}$ or E_k) and rotational kinetic energy ($E_{rotational}$ or E_r) it can be shown that the proportion of potential energy converted into linear and rotational kinetic energy is fixed by a ratio when there is no slipping. We see, that independent of the radius or mass of the cylinder,

$$\left.\begin{aligned} E_k &= \frac{2}{3}E_p \\ \text{and} \quad E_r &= \frac{1}{3}E_p \end{aligned}\right\} \therefore \; E_k : E_r = 2:1$$

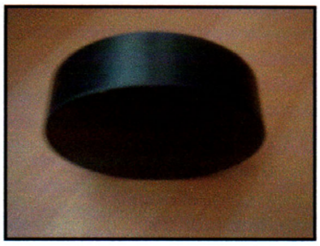
(check appendix 6 on page 23 for the full development of the equations).

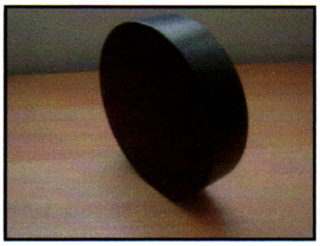
The theory tells us that this 2:1 ratio should always hold true. To verify if this is still the case under slipping conditions, as well as to verify our previous predictions, we need to collect experimental data and reach results. The next part of this essay therefore focuses on the experiments performed and the analysis of the data collected.

3. Experiment Setup Description:

An experiment is designed to study the motion of a standard cylinder placed on an inclined plane. The cylinder we used is made of smooth plastic, and is shown in photo 1 and 2 below. Its radius measures 15.2 cm ± 0.05 cm and its weight is 652.1 g ± 0.05 g.

Photo 1 and 2: the standard cylinder:

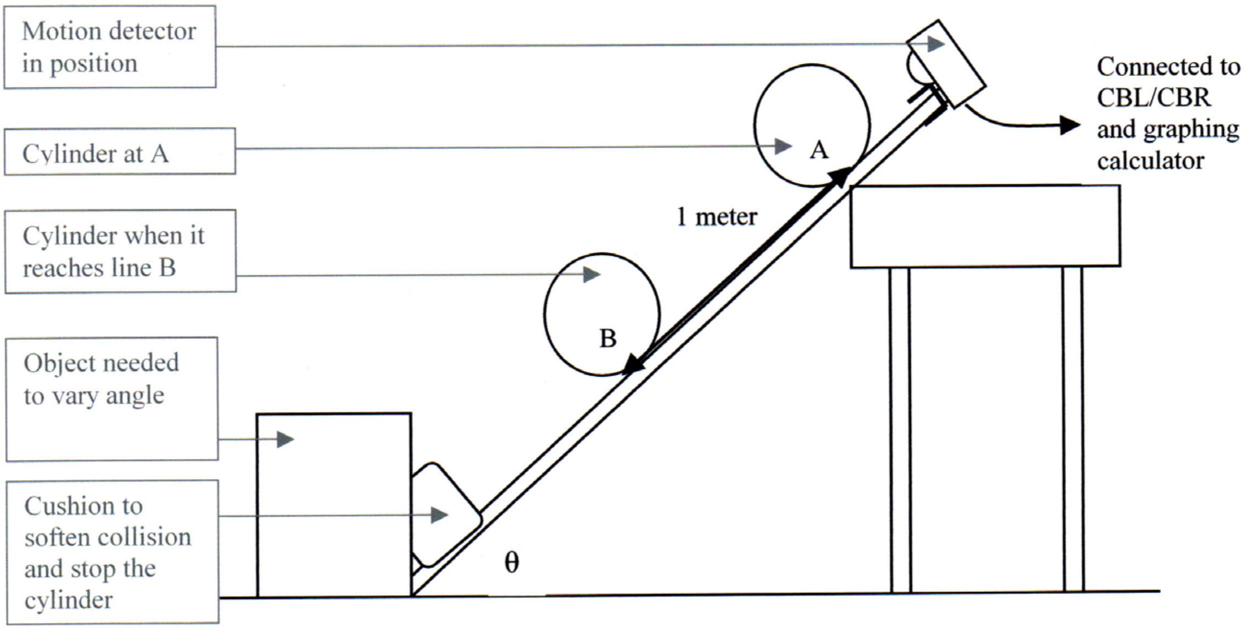




1 cm on photo = 6 cm real size

The experiment also requires the use of an inclined plane. For the experiment, a smooth wooden board is used for the plane. The board is segmented by two marks, which are labeled A and B. It is also important to note that the wooden board's height can be adjusted to vary the angle of elevation θ. The purpose of this experiment is to determine the linear velocity of the cylinder when it crosses line B. Diagram 2 below represents the experiment setup and apparatus used:

Diagram 2: The apparatus and setup:



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4. Measuring Instrument Description:

The instruments used to collect data include: a digital motion detector, a graphing calculator, a CBL/CBR unit and a protractor.

As we can see in the diagram above, the motion detector is placed on the edge of the board in order to collect measurements along and down the plane. It is an ultrasonic device that works by sending waves and collecting the reflection of those waves off surfaces in its line of action. It then processes this information and records a displacement value for the cylinder. The increment of time between each measurement can be manually adjusted along with the duration of the sample taking. It becomes obvious that because we are interested in the linear velocity at B ($v = \frac{\Delta s}{\Delta t}$), the smaller Δt is, the more accurate the instantaneous velocity at B becomes. Therefore for the data collection the time interval is always set to be as small as possible. There is, however, a limit to how small Δt can be because the motion detector then easily 'saturates' and produces noise data when Δt is small.

The CBL/CBR unit and the graphing calculator are necessary in the experimental setup to transfer the data from the motion detector to the computer. When the data is collected, it is then retrieved with the help of these devices onto computer software – graphical analysis 3.4 – (Program cited on page 26) and analyzed. To sum it up, the combination of the motion detector with the calculator and CBL/CBR measures displacements and calculates (as an approximation) instantaneous linear velocities.

The last instrument used in the experiment is a protractor. Since it measures angles with a limit of reading of 1°, the uncertainty with each angle value is ±0.5°. In the work of this essay, the span of angles investigated ranges from 5° to 70° with 5° increments each time. The investigation does not cover angles beyond this range because it is impractical to conduct the experiment at angle lower than 5° and larger than 70°. Likewise taking measurements on a smaller increment basis is also unfeasible because of the limitations with measuring angles and time factor.

5. Calibration and accuracy tests:

Before the collecting the data, it is important to check if the motion detector is reliable and if so, how reliable. Two main tests are performed to prepare for the main experiment of this essay.

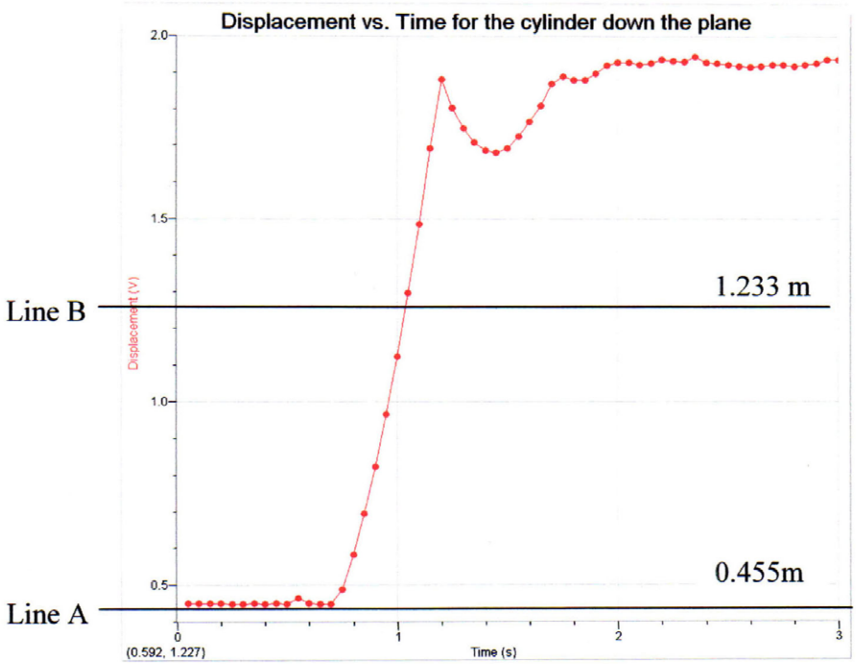
First we need to make sure that the motion detector works well enough and is accurate enough for our purpose. For this, the cylinder is placed on line B, the motion detector is started and twenty distance measurements of the cylinder's position are taken. The readings are recorded. The average and the widest scatter from average are then calculated to determine the accuracy. The latter turned out to be quite satisfactory as the average of the measurements was 1.233± 0.002 meters (check appendix 1 for the data on page 13): this gives reasonable evidence that the motion detector can determine position almost down to the single millimeter.

Secondly, another test is done to check that the velocity calculations of the detector are reliable. A low angle is chosen, 5°, to avoid slipping, and the instantaneous velocity value of the cylinder at B is calculated for ten measurements. The average and widest scatter are again recorded. The results are: 1.042 ms^{-1} ± 0.014 ms^{-1}, showing that the motion detector's accuracy is still quite acceptable on that level as well (measurements can keep two decimal places).

6. Experiment Procedures and Recorded Results:

The idea in this experiment is to vary the angle θ and to determine the linear velocity v of the cylinder at line B. First the smooth wooden board is set into place and its slope is adjusted to arrive at the desired angle. The CBL/CBR and calculator are then set to collect data and connected to the motion detector. The cylinder is carefully placed on line A along the axis of the motion detector in such way that it will roll down when released. Then the motion detector is manually started, leaving the cylinder into place at A for a brief moment. This way we can see whether or not the cylinder was moving before its release. Then the cylinder is lightly released down the plane. Data is collected and then retrieved onto Graphical Analysis 3.4, which puts together displacement vs. time graphs. A sample data graph from this program is reproduced in graph 1 below.

Graph 1: Sample data with 50° incline: Finding the point where line B is crossed:



We can see that the cylinder picks up speed when it is released (when the displacement readings start to increase drastically). We also know from the previous accuracy tests that the cylinder crosses line B when the displacement reads 1.233m. However, our data points might not fall exactly on 1.233m. It might fall *in between* two data points. By using a simple rule of proportion (a more detailed explanation of this process is found in appendix 2 on page 14), we can interpolate to find the time taken t_B by using two neighboring points. Then knowing t_B for each set of data, we can use the velocity vs. time graphs generated by the program (which simply finds the slope between each consecutive point) to get a value for the linear velocity of the cylinder at B.

Finding the slope between each consecutive data point allows drawing a velocity vs. time graph. A sample is shown below in graph 2. From the time the cylinder is dropped to the time it stops, the graph shows a linear relationship. This makes sense because in theory, the acceleration down the plane caused by gravity is constant at any given angle ignoring friction effects (Hecht 327). Hence the velocity should increase

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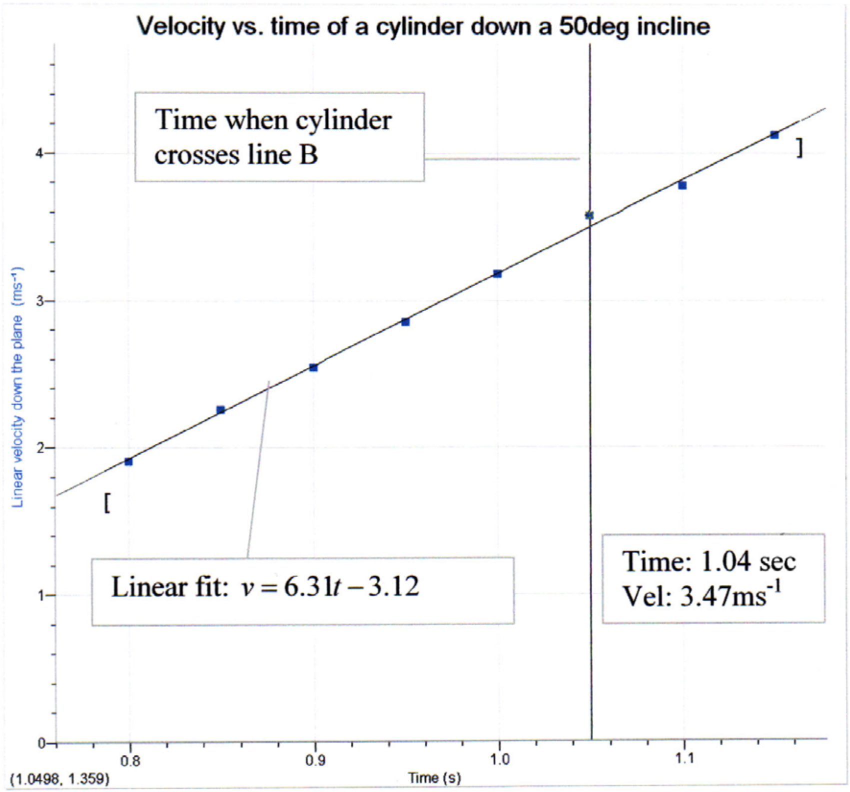
uniformly, which is what we see. By drawing a line of best fit, we can take an overall average of the relationship. This corrects, for example, random errors which may occur if for some reason the motion detector produces an 'odd' point at the points we are interested in. Knowing t_B from the work we did with the displacement vs. time graphs, we can interpolate the regression and find the desired velocity.

Graph 2: Sample data with 50° incline:
How to find the velocity at point B:



7. Using Theory to calculate linear velocity (v):

Now that we know a procedure to determine the experimental velocity for any angle, it is then interesting to calculate the theoretical velocities at each of these angles and use them for comparisons later on.

After using the energy theorem and simplifying (check appendix 4 for the full development of the equation on page 20), we finally solve for the linear velocity v_B at B expected at any angle ranging from 0° to 90°. This relation is as follow:

$$v_B = \sqrt{\frac{4}{3} g \sin\theta} \qquad [1]$$

where θ is the angle of elevation of the plane, g the gravitational acceleration 9.81 ms^{-2}. This equation assumes that no slipping occurs, and that for the cylinder: $v = r\omega$. This is crucial to the analysis, as we will see later on.

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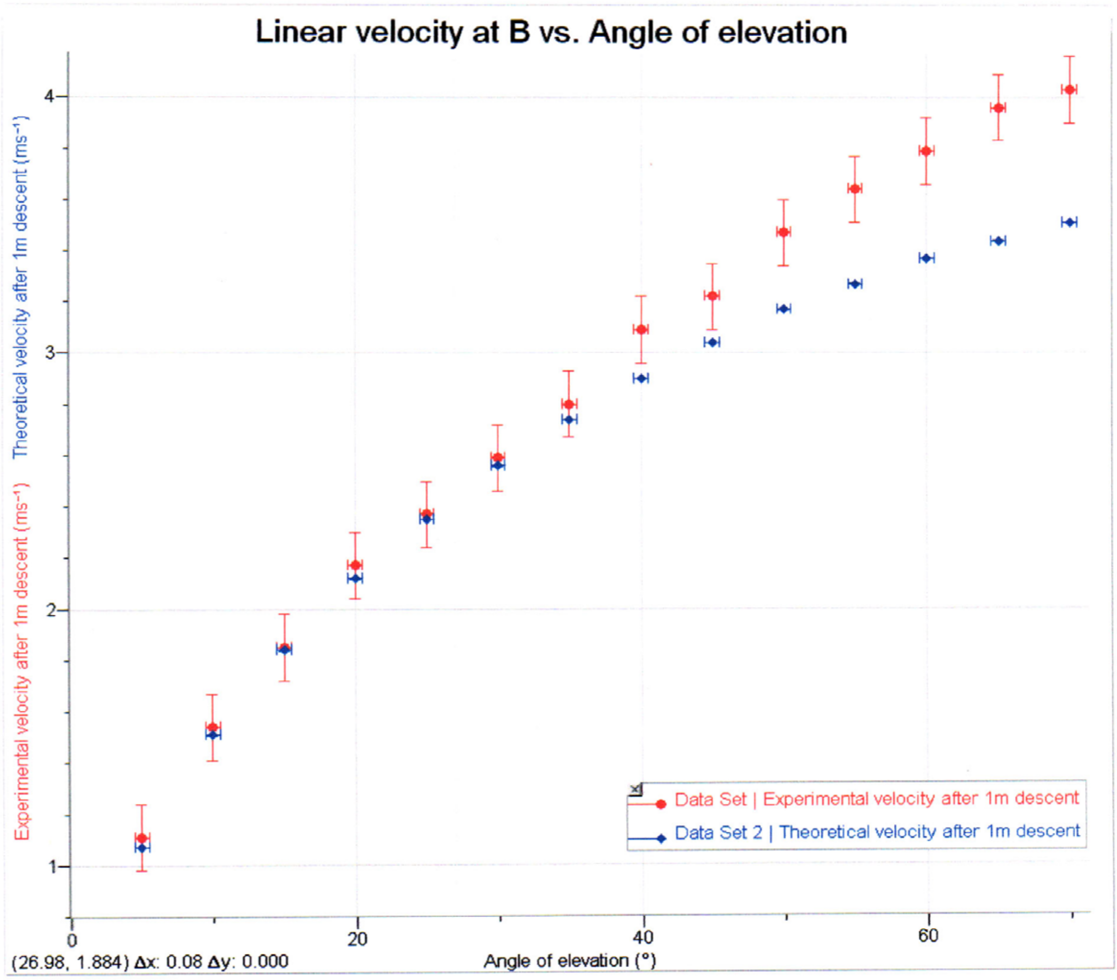
The process of collecting velocity-data is then done for fourteen measurements (5° up to 70°) with the smooth wooden board and our standard cylinder. In addition to this, the theoretical value for each of these angles is found by simply substituting the relevant angle value in equation [2] above. The full list of data plots is shown in appendix 3 on page 19.

8. Data Analysis:

The results found above are then plotted on a linear velocity vs. angle graph, as illustrated below in graph 3.

Graph 3: Data for the standard cylinder with theoretical values:



Two immediate observations follow from this graph: firstly, that both plots seem to follow a slightly curved pattern, flattening towards the end in both case (showing that the relationship is a square root function). Secondly, that the experimental points follow closely the theoretical ones until a certain point. The second observation is the one that matters most in our investigation. From the angle value of 5° to about 35°, both data plots

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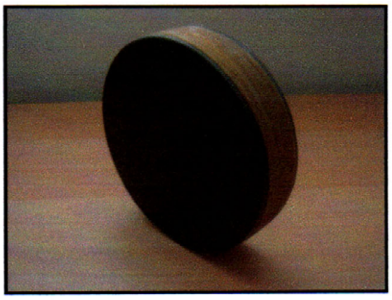
overlap each other rather well; the theoretical data even fits within the uncertainty bars. However past 35°, the experimental velocity increasingly surpasses the theoretical velocity. This seems odd indeed since it is expected that both line intertwine all the way. The most logical reason behind this observation is that slipping occurs. As a result, the assumption made in the initial theoretical model, that is, $v = r\omega$, does not hold anymore.

However, it is important we confirm that it is slipping which is responsible for this anomaly. We may not immediately jump to this conclusion. At this stage, we are not certain of this and the difference may be caused by some other factor. Therefore a second but similar experiment is repeated with the cylinder adjusted slightly, as explained in the next section.

9. Verification that the graphical difference is caused by slipping:

To verify that the disparity between both sets of data is actually caused by slipping, a rubber surface is fitted onto the cylinder to increase its grip on the board. This is done by assembling multiple elastic bands around it. This has the effect of raising substantially the friction between the plane and the cylinder (while the mass and radius increase of the cylinder are minute, on the other hand). A photographic illustration of the abovementioned cylinder is represented below in photo 3 and 4:

Photo 3 and 4: the standard cylinder wrapped with rubber bands:



1 cm on photo = 6 cm real size

The underlying assumption made here is that by substantially increasing the friction between the cylinder and the board, slipping will be reduced to a minimal.

After conducting the same experiment as before, we collected a full data set and graphed the data. The data collected may be examined in appendix 7 on page 24. The graph is shown below in graph 4:

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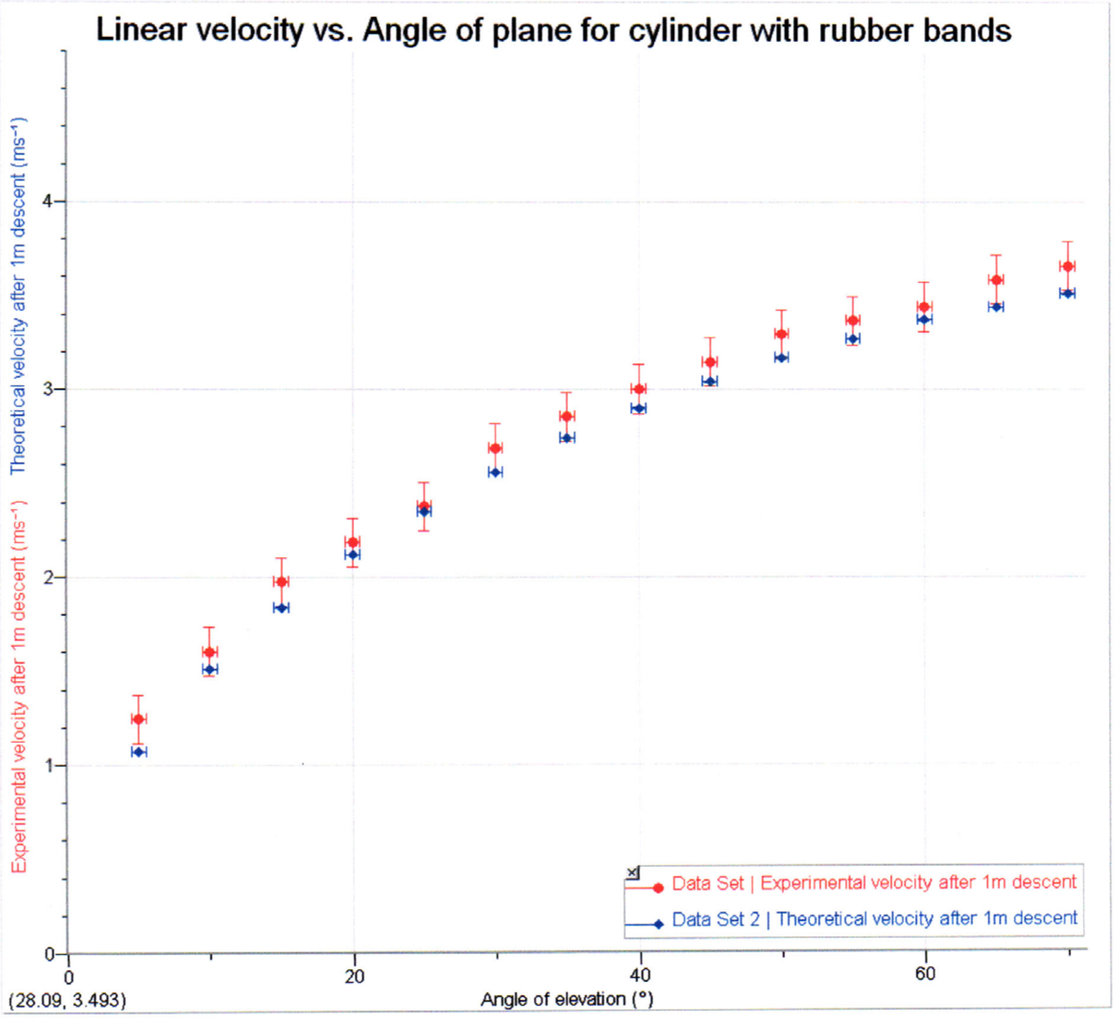
<u>Graph 4: Data for the standard cylinder with rubber bands:</u>



We can see from this graph that both the experimental and theoretical data set overlap each other almost perfectly. This graph confirms that slipping is responsible for the disparity between each set in graph 3. The graph also shows evidence of a systematic error of about $0.1ms^{-1}$ higher for the experimental plot. This is most likely due to the way the motion detector was set in place, since we did not use the motion detector clamp for this part of the experiment (a change which should not have occurred) which could have added a bit of distance. The next part of this essay now looks at finding the angular velocity of the cylinder for each angle value.

10. Determining the Angular Velocity (ω):

It is interesting then to look at what happens to the angular velocity at B and as the angle of elevation is raised. This can be derived from theory. Coming back to the energy theorem and this time solving for ω in terms of the linear velocity found before (yet still making the $v = r\omega$ assumption), we can find the angular velocity that the cylinder has for each angle we experimented with. This again assumes that there is no loss of energy due to friction. Furthermore, since the velocity data collected before has

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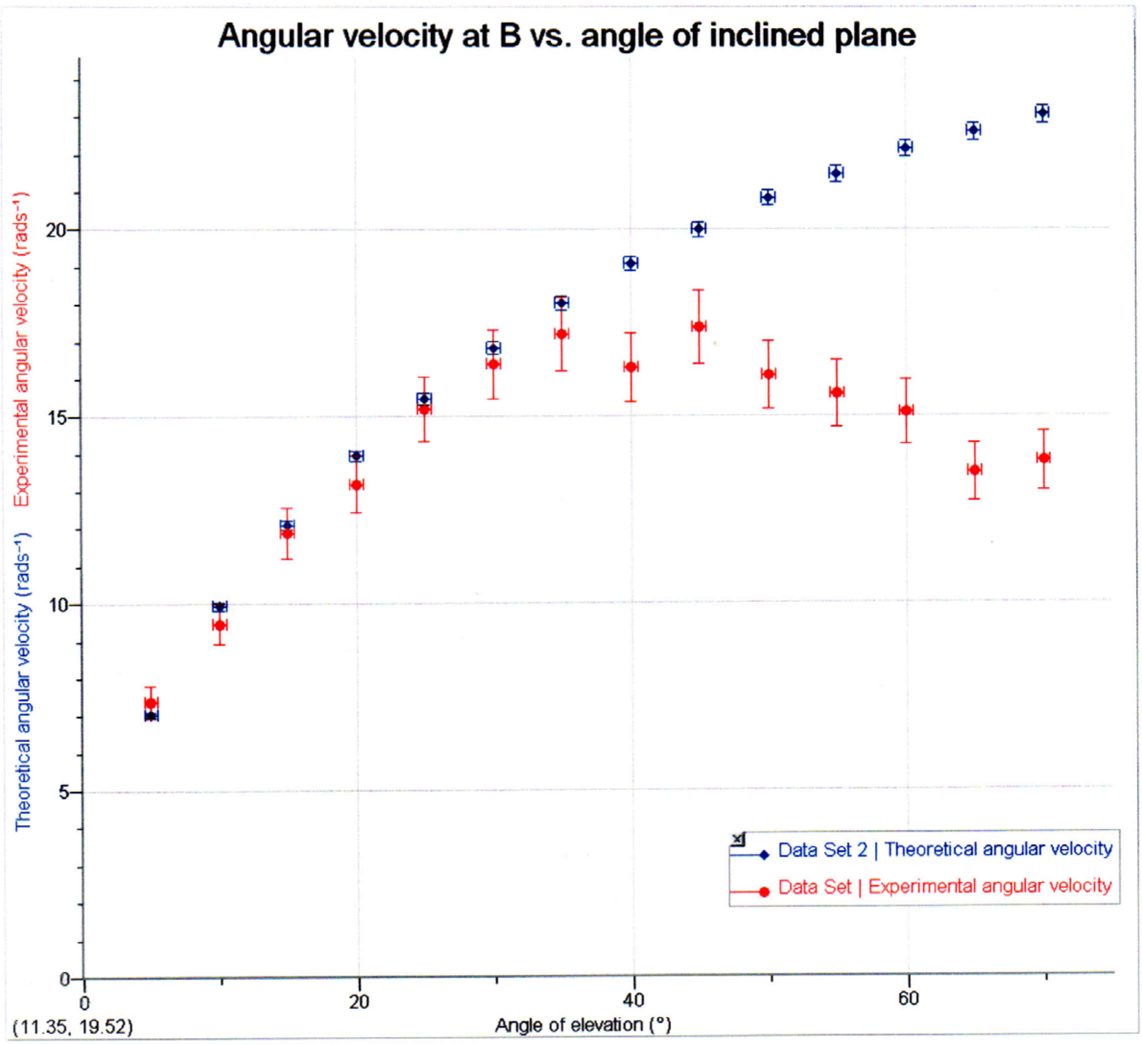
been processed already, reprocessing processed data will deteriorate the accuracy of the angular velocities measurements. It is for that reason that the angular velocity data is used more as a guideline to understand the phenomenon of slipping, rather than accurate data used for more thorough investigation. It would have been interesting, if not necessary, to determine the angular velocity of the cylinder by experiment rather than theory (as what is being done here); however due to lack of time and adequate apparatus, an experimental method could not be carried through for this essay. The development of equations used to arrive to the final equation [2] below is found in appendix 5 on page 22, and yields the following relation:

$$\omega_B = \frac{\sqrt{4g\sin\theta - 2v_B{}^2}}{r} \qquad [2]$$

where v_B is the linear velocity at B, r is the radius of the cylinder and θ is the angle of the inclined plane. After measuring the radius of the cylinder with an accurate ruler, we get the following result: Cylinder radius: 0.152 m ± $5\cdot10^{-4}$m. We then generate data by substituting the relevant values for θ and v. The data is found in appendix 3 on page 19. We get the results found in graph 4 below:

Graph 4: Data for the angular velocity with theoretical values:



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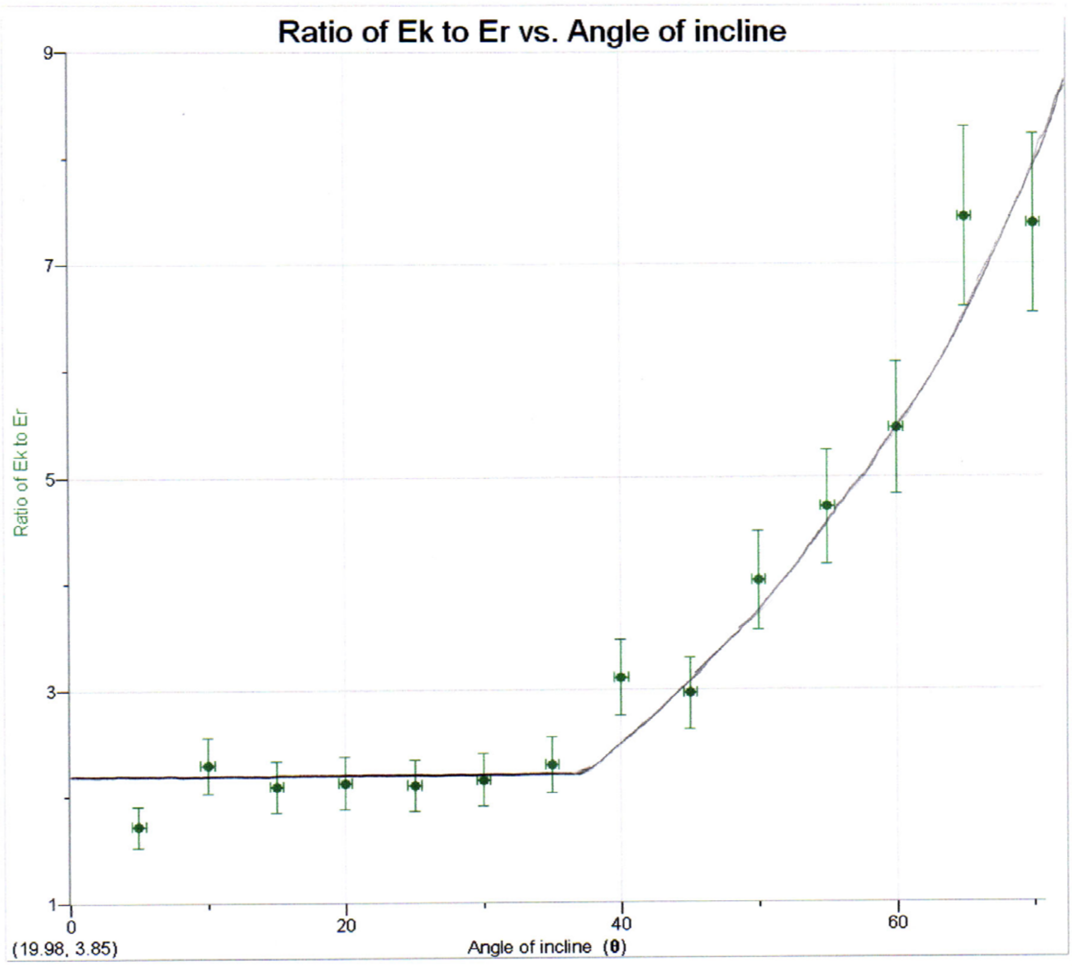
Compared to the previous graph, this one shows rougher trends. Moreover the theoretical values barely, if at all, fall within the error bars, which suggests that the points are not very accurate. Yet there is a distinct trend for both the experimental and theoretical plot. Although the theoretical data follows a smooth curved shape, the experimental pattern resembles more a negative parabola. The angular velocity indeed drops after a peak located at about 30-35°. Because this range matches the beginning of slipping in the previous graph, we can deduce that during slipping the angular velocity is reduced to the expense of higher linear velocity.

11. Finding the energy ratio during slipping:

Referring to our second objective, we are now interested in finding how the ratio of energy in linear velocity to energy in angular velocity varies when there is slipping or not. Linear kinetic energy is given by the equation: $E_k = \frac{1}{2}mv^2$. Knowing m and v, we can thus calculate E_k. Likewise, rotational kinetic energy is given by the equation: $E_r = \frac{1}{2}I\omega^2$. Knowing ω and I ($I = \frac{1}{2}mr^2$ for a cylinder, which is known (Hecht 323)), we can calculate E_r. The generated data for the ratio of E_k to E_r is found in appendix 8 on page 25. It is illustrated below:

Graph 5: Data for the ratio of energies:



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We can distinguish two distinct sections in this graph. First, there is the range 5°-35° where the trend seems to be a flat linear pattern anchored around the ratio of 2. This coincides with the theoretical value when there is no evident slipping as the proportion of energy is divided in two-third part as linear kinetic energy and one-third part as the rotational kinetic energy, giving a ratio of two ($E_k : E_r = 2:1$). Secondly, there is the range 35°-70°, where the ratio escalates rapidly. It represents the part where slipping becomes significant. We can therefore see that the ratio increases with slipping and, apparently, at an increasing rate (we must remember that at 90°, the ratio would be very large as E_r approached 0). The nature of the error bars shows that earlier points can be trusted more than later points. Yet the graph still provides a reasonably accurate picture of the overall trend of the ratio over the angle intervals.

12. Evaluation and Conclusion:

This investigation has led to several interesting observations.

First of all, we noticed that the phenomenon of slipping occurred predominantly at large angles of elevation and in for our experimental setup, slipping became noticeable graphically at around 35-40° (it may have started earlier than that but not have been shown distinctly on the graphs).

Secondly, we noticed that during slipping, the linear velocity increased quite significantly. Again since the velocity measurements were quite precise (as verified by one of the calibration test performed), we are fairly certain that the difference between the experimental and theoretical data in graph 3 is not due to experimental errors, whether it be systematic or random, but in fact due to slipping. We have verified this with yet another setup using elastic bands on the cylinder; and the results from graph 4 confirm that slipping could have caused the increase in linear velocity.

Angular velocity data shows, on the other hand, that the angular velocities dropped after a particular angle range, similar to that of graph 3 (range of 35-40°), again suggesting that slipping was responsible for that drop.

Relating to our second objective, the energy ratio, which was supposed to hold true regardless of the angle range, also seems to break down when slipping occurs. As we expected, the ratio of 2:1 linear K.E. to rotational K.E. worked for low angles (5-35°). But when the cylinder slipped significantly, the ratio increased at a rapid rate, climbing up to 4:1 for 50° and 7:1 for 70°.

Therefore overall the observations made still permit us to draw a conclusion on the motion of the cylinder as it slips and rolls down the plane. We now have greater certainty (even though we may not jump to the conclusion that such results is scientific knowledge as such) that when the cylinder slips, it naturally rolls less about its center of gravity and let it translate more. The argument we give to explain this is that because the force of friction drops as the angle is raised, it cannot provide a large enough torque to keep making the cylinder roll as much as expected by theory for large angles.

There are certain uncertainties in some of the work of this essay. Part of this uncertainty is due to the lack of measurements taken. For example, instead of calculating the angular velocity directly experimentally, we used the linear velocity measurements and the energy theorem to derive it, which brought more uncertainty within the data. This investigation would have been stronger if we had collected first-hand angular velocity data. Secondly, only one complete set of data is presented here. Repeating the investigation a number of times would increase the certainty of the results and make this investigation stronger. Thirdly, the setup itself produced errors in the measurements. For example, if the cylinder was not placed exactly perpendicular to the edge of the board, its path would have deviated sideways and the cylinder would have traveled more than one meter when reaching line B. Also, it is possible that due to its curved surface, echoes from the motion detector may have reflected off both the cylinder and the board, giving a longer displacement measurement than expected (this might also be a reason for the systematic errors observed previously).

Quite a few questions emerge from this investigation. It would have been interesting to test different surfaces and see how the maximum angle before slipping changes to attempt to generalize, or to repeat the experiment for solids such as spheres, cubes, rings, etc…for comparison. In addition, it would also have been worthwhile to continue further the analysis to acquire a quantitative measure of the degree of slipping for the setup, which hasn't been done so far.

One major unresolved question in this essay came with the observation that line A and line B, which are in actual fact 1.000 meter apart, appear not to be according to the motion detector (B-A = 1.233m - 0.455m ≠ 1.000m). The most probable reason we came with was that the conversion from a voltage reading to a distance reading uses a scale that is a bit off, thus explaining that discrepancy; but this is yet only a suggestion. However since this effect has been occurring throughout all the experiments, there is fair certainty to say it has not affected the results as such.

There are several applications arising from this investigation. One suggestion would be to use similar procedures to test the degree of slipping in bodies such as automobile wheels. If a similar experiment was to be repeated along those lines, we could devise a model of a mobile cart and change certain parameters, such as load in the cart, tire type, design of the cart, etc… while measuring the degree of slipping each time, using the procedures we followed in this investigation. Such an investigation would surely produce worthwhile results.

Word Count: 3968.

13. Appendix 1:

First calibration test:

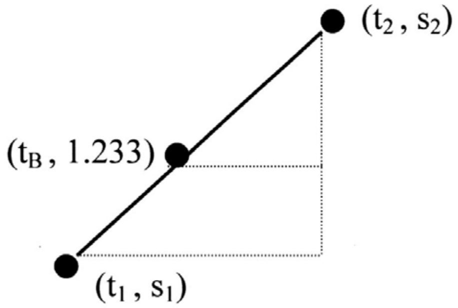
Measurements taken for line B (m)					Average of the measurements (m)	Widest increment away from average (m)
1.233	1.232	1.235	1.233	1.233	1.233	±0.002
1.233	1.233	1.233	1.234	1.233		
1.234	1.234	1.233	1.233	1.233		
1.233	1.233	1.233	1.233	1.233		

Second calibration test:

Measurements taken for instantaneous velocity at B (ms^{-1})					Average of the measurements (ms^{-1})	Widest increment away from average (ms^{-1})
1.050	1.056	1.043	1.040	1.042	1.042	± 0.014
1.035	1.035	1.036	1.042	1.037		

14. Appendix 2:

Let us consider the representation below of two consecutive points collected by the detector where (t_B , 1.233) is in between:



Assuming that such points are arranged on a line, then from the rule of proportion we have:

$$\frac{(s_2 - s_1)}{(t_2 - t_1)} = \frac{(s_2 - 1.233)}{(t_2 - t_B)}$$

$$(t_2 - t_B) = \frac{(s_2 - 1.233)(t_2 - t_1)}{(s_2 - s_1)}$$

$$t_2 - \frac{(s_2 - 1.233)(t_2 - t_1)}{(s_2 - s_1)} = t_B$$

$$t_B = t_2 - \frac{(s_2 - 1.233)(t_2 - t_1)}{(s_2 - s_1)}$$

This is the time at which the cylinder crosses B, where (t_1 , s_1) are the coordinates of the data point below (t_B , 1.233) and where (t_2 , s_2) are the coordinates of the data point above (t_B , 1.233). This equation was used to get a more accurate value for t_B in the analysis.

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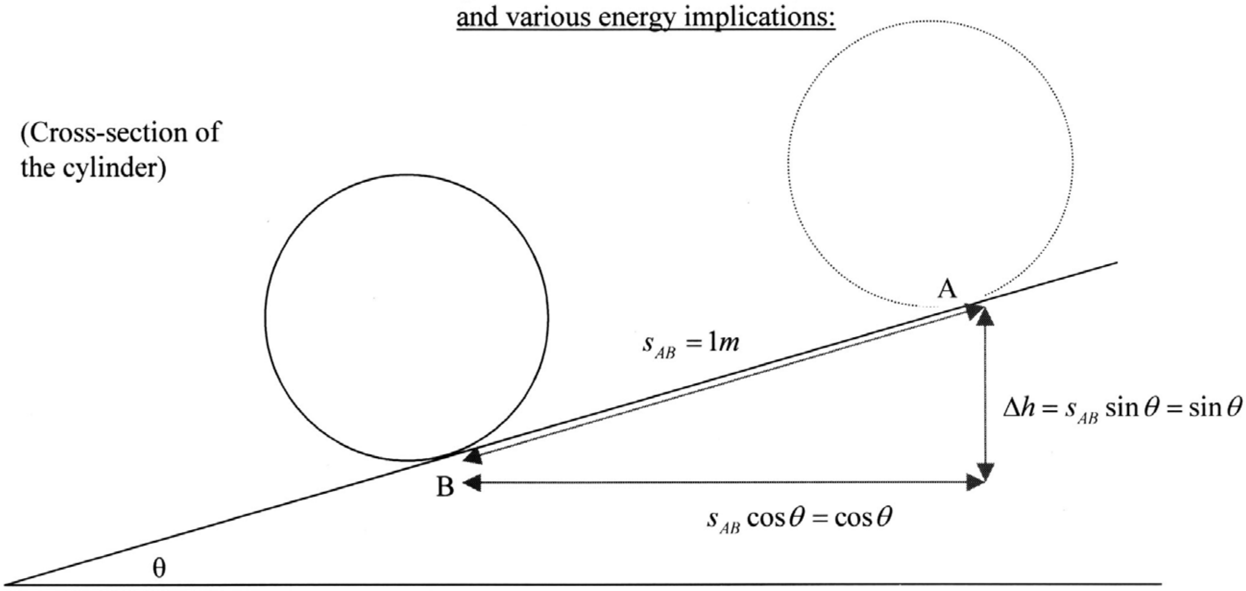
15. Appendix 3:

Angle (θ) ± 0.5°	Experimental linear velocity ± 0.13 ms^{-1}	Theoretical linear velocity ± 0.05 ms^{-1}	Experimental angular velocity ± 5.7% $rads^{-1}$	Theoretical angular velocity ± 0.05 $rads^{-1}$
5	1.04	1.07	7.38	7.30
10	1.54	1.51	9.46	9.54
15	1.85	1.84	11.9	11.64
20	2.07	2.12	13.2	14.21
25	2.37	2.35	15.2	15.26
30	2.59	2.56	16.4	17.24
35	2.80	2.74	17.2	19.21
40	3.09	2.90	16.3	20.92
45	3.22	3.04	17.4	22.04
50	3.47	3.17	16.1	23.62
55	3.64	3.27	15.6	25.07
60	3.79	3.37	15.1	25.72
65	3.96	3.44	13.5	26.91
70	4.03	3.51	13.8	27.17

16. Appendix 4:

Over its descent, the cylinder drops a height Δh. The displacement down the plane, s_{AB}, was chosen to be 1 meter exactly to simplify calculations. The situation is modeled in diagram 3 below:

Diagram 3: The descent of the cylinder and various energy implications:



The cylinder has thus lost gravitational potential energy, of an order of: $E_{potential} = mg\Delta h$. This energy will be transferred in three main forms: in translational kinetic energy: $E_{kinetic} = \frac{1}{2}mv^2$ (Hecht 237), in rotational kinetic energy: $E_{rolling} = \frac{1}{2}I\omega^2$ (Hecht 329) and in energy lost to friction. When the motion consists only of pure rolling, there is very little energy loss due to friction (rolling friction is not dissipative). The fact that in rolling the force of friction does an infinitesimal amount of work means that we can also ignore it. If slipping occurs there will be some energy lost to friction. However, because the experiment occurs over the span of 1m and involves smooth materials (plastic and polished wood), that amount is likely to be small, and will be ignored. After the principle of the conservation of energy, we have the following equation:

$$\Delta E_{potential} = \Delta E_{kinetic} + \Delta E_{rolling}$$

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgs_{AB}\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$$

Assuming no slipping occurs; the assumption $v = r\omega$ or $\omega = \frac{v}{r}$ holds true.

$$mg\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v^2}{r^2}\right)$$

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$$mg\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right)$$

← For a cylinder or a disk, the moment of inertia I is equal to $\frac{1}{2}mr^2$.

$$mg\sin\theta = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$mg\sin\theta = v^2\left(\frac{1}{2}m + \frac{1}{4}m\right)$$

$$v^2 = \frac{mg\sin\theta}{\frac{3}{4}m}$$

← Then solving for v

$$v^2 = \frac{4}{3}g\sin\theta$$

$$v = \sqrt{\frac{4}{3}g\sin\theta}$$

← This equation gives the linear velocity that we expect at B based on the angle of the inclined plane (assuming pure rolling).

17. Appendix 5:

Determining ω from theory as a function of the linear velocity *v*, the cylinder radius *r* and the angle of elevation of the plane *θ*:

$$\mathrm{mg}\Delta h = \frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{I}\omega^2$$

For a cylinder or a disk, the moment of inertia I is equal to $\frac{1}{2}mr^2$.

$$\mathrm{mgs}_{\mathrm{AB}}\sin\theta = \frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}(\frac{1}{2}mr^2)\omega^2$$

$$\mathrm{mg}\sin\theta = \frac{1}{2}m\left(v^2 + \frac{1}{2}r^2\omega^2\right)$$

Factorizing $\frac{1}{2}$m from the expression.

$$2g\sin\theta = v^2 + \frac{1}{2}r^2\omega^2$$

$$2g\sin\theta - v^2 = \frac{1}{2}r^2\omega^2$$

Solving for ω in terms of v and r.

$$4g\sin\theta - 2v^2 = r^2\omega^2$$

$$\omega^2 = \frac{4g\sin\theta - 2v^2}{r^2}$$

$$\omega = \frac{\sqrt{4g\sin\theta - 2v^2}}{r}$$

With this equation in mind, we can determine the theoretical values that we should expect (within reasonable uncertainty ranges) were we to carry out the experiment.

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18. Appendix 6:

Starting from the energy theorem (energy conservation), we reach the following two conclusions for non-slipping conditions:

Linear kinetic energy:

$$E_{potential} = E_{kinetic} + E_{rolling}$$

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$$

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v^2}{r^2}\right)$$

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right)$$

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$mg\Delta h = \left(\frac{3}{2}\right)\frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = \frac{2}{3}mg\Delta h$$

$$E_{kinetic} = \frac{2}{3}E_{potential}$$

Rotational kinetic energy:

$$E_{potential} = E_{kinetic} + E_{rotational}$$

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mg\Delta h = \frac{1}{2}m(\omega r)^2 + \frac{1}{2}I\omega^2$$

$$mg\Delta h = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}I\omega^2$$

$$mg\Delta h = I\omega^2 + \frac{1}{2}I\omega^2$$

$$mg\Delta h = 3\left(\frac{1}{2}I\omega^2\right)$$

$$\frac{1}{2}I\omega^2 = \frac{1}{3}mg\Delta h$$

$$E_{rotational} = \frac{1}{3}E_{potential}$$

These two equations explain how, no matter what the linear or angular velocity is, the gravitational potential energy used up in the cylinder's descent will always split up in a ratio of $\frac{1}{3}$ rotational K.E. to $\frac{2}{3}$ linear K.E.

$$\therefore \quad E_k : E_r = 2:1$$

19. Appendix 7:

Data for cylinder with rubber bands:

Angle (θ) ± 0.5°	Experimental linear velocity ± 0.13 ms^{-1}	Theoretical linear velocity ± 0.05 ms^{-1}
5	1.24	1.07
10	1.61	1.51
15	1.98	1.84
20	2.18	2.12
25	2.38	2.35
30	2.69	2.56
35	2.85	2.74
40	3.00	2.90
45	3.15	3.04
50	3.29	3.17
55	3.37	3.27
60	3.44	3.37
65	3.59	3.44
70	3.66	3.51

20. Appendix 8:

Processed data for linear and rotational K.E. as well as energy ratio:

Angle (θ) ± 0.5°	Linear kinetic energy stored ($0.5mv^2$) Joules (±5.7%)	Rotational kinetic energy stored ($0.5I\omega^2$) Joules (±5.7%)	Ratio of E_k to E_r (±11.4%)
5	0.353	0.205	1.719
10	0.773	0.337	2.294
15	1.116	0.533	2.092
20	1.397	0.656	2.129
25	1.831	0.870	2.105
30	2.187	1.013	2.159
35	2.556	1.114	2.294
40	3.113	1.001	3.111
45	3.381	1.140	2.965
50	3.926	0.976	4.021
55	4.320	0.917	4.713
60	4.683	0.859	5.453
65	5.113	0.686	7.448
70	5.295	0.717	7.382

21. Works and Program Cited:

Program:

Graphical Analysis 3.4
Sept 26 2005
Copyright (c) 2005 Vernier Software & Technology

- Graphical Analysis is the basic graphing and analysis software from Vernier Software Technology. It combines graphing, TI calculator data import, curve fitting, and other analytical tools into one easy-to-use program.

Book Resources:

Hecht, Eugene. Physics: Calculus. 2nd ed. Pacific Grove, CA: Thomson Learning, 2000.

