Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

1 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2224 – 7201

25 pages

Please **do not** write on this page.

Answers written on this page
will not be marked.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Maan deposited $\$100\,000$ into a savings account with a nominal annual interest rate of $I\,\%$ **compounded monthly**. At the end of the eighth year, the amount in the account had increased to $\$150\,000$.

(a) Find the value of I. [3]

Maan withdraws the $\$150\,000$ and places it in an annuity, earning a nominal annual interest rate of $6.1\,\%$ **compounded monthly**. At the end of each month, Maan will receive a payment of $\$1000$.

(b) Find the amount of money remaining in the annuity at the end of 10 years. Express your answer to the nearest dollar. [3]

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2. [Maximum marks: 7]

After taking a mathematics test, Fatima wonders how many more marks she would have achieved if she had spent an extra 1.5 hours studying.

To find out, she randomly selects five students from her class who took the same test and asks them how many hours (t) they spent studying for the test and the marks (m) they achieved. Their responses are shown in the following table.

Hours, *t*	0	1.2	1.6	2.5	4
Marks, *m*	45	54	61	72	86

Fatima believes there might be a linear relationship between the time spent studying and the results obtained.

(a) (i) Find the Pearson's product moment correlation coefficient, r, for this data.

(ii) Find the least squares regression line of m on t for this data. [4]

(b) According to her model, find how many more marks Fatima would have achieved if she spent an extra 1.5 hours studying. [2]

(c) State one reason why the value obtained in part (b) might not be valid. [1]

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3. [Maximum mark: 6]

The formula $F = 1.8C + 32$ is used to convert a temperature in degrees Celsius, C, to degrees Fahrenheit, F.

(a) (i) Find a formula for converting a temperature in degrees Fahrenheit to degrees Celsius.

(ii) Find the temperature in degrees Celsius that is recorded as 77 degrees Fahrenheit. [3]

Over one year, the mean daily temperature in Mexico City was calculated to be 17 degrees Celsius with a standard deviation of 9 degrees Celsius.

(b) For the same year, find in degrees Fahrenheit

(i) the mean daily temperature in Mexico City.

(ii) the standard deviation of the daily temperature in Mexico City. [3]

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4. [Maximum mark: 4]

Consider the differential equation $\frac{dy}{dx} = \log_{10}(x + y)$, where $x \geq 0$ and $y > 0$.

Given that $y = 1$ when $x = 0$, use Euler's method with a step length of 0.1 to find an approximate value for y when $x = 2$.

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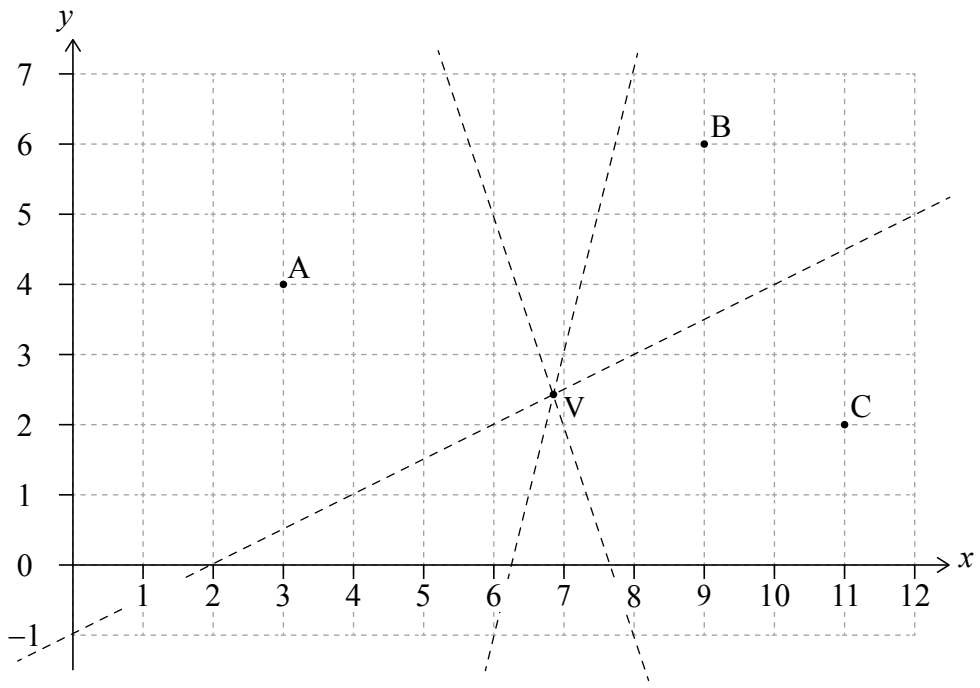
5. [Maximum mark: 6]

Points $\mathrm{A}(3,\ 4)$, $\mathrm{B}(9,\ 6)$ and $\mathrm{C}(11,\ 2)$ are shown on the following diagram, along with the perpendicular bisectors of [AB], [AC] and [BC].



The perpendicular bisector of [BC] intercepts the axes at coordinates $(0,\ -1)$ and $(2,\ 0)$.

(a) Write down the equation of the perpendicular bisector of [BC]. [2]

The equation of the perpendicular bisector of [AB] is $y = -3x + 23$.

(b) Find the coordinates of point V where the perpendicular bisectors meet. Give your answer to four significant figures. [2]

A Voronoi diagram is constructed with points A, B and C as the three sites.

(c) Draw, clearly, the edges of the Voronoi diagram on the given diagram. [2]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 7]

On 1 January 2025, the Faber Car Company will release a new car to global markets. The company expects to sell 40 cars in January 2025. The number of cars sold each month can be modelled by a geometric sequence where $r = 1.1$.

(a) Use this model to find the number of cars that will be sold in December 2025. [2]

(b) Use this model to find the total number of cars that will be sold in the year

(i) 2025.

(ii) 2026. [5]

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7. [Maximum mark: 8]

When studying big cats, researchers use a model in which the mass (m kilograms) of an animal is directly proportional to the cube of its shoulder height (h metres).

Cheetah

Lion

A cheetah has a mass of $64\,\text{kg}$ and shoulder height of 0.8 metres.

(a) (i) Use the model to find an expression for m in terms of h.

(ii) Hence find the mass of a different cheetah, with a shoulder height of 0.75 metres. [4]

'Rubner's law' states that the energy needs of an animal (E) are directly proportional to the square of h.

The energy needs of a lion of mass $220\,\text{kg}$ are k times the energy needs of a cheetah of mass $64\,\text{kg}$.

(b) Find the value of k. [4]

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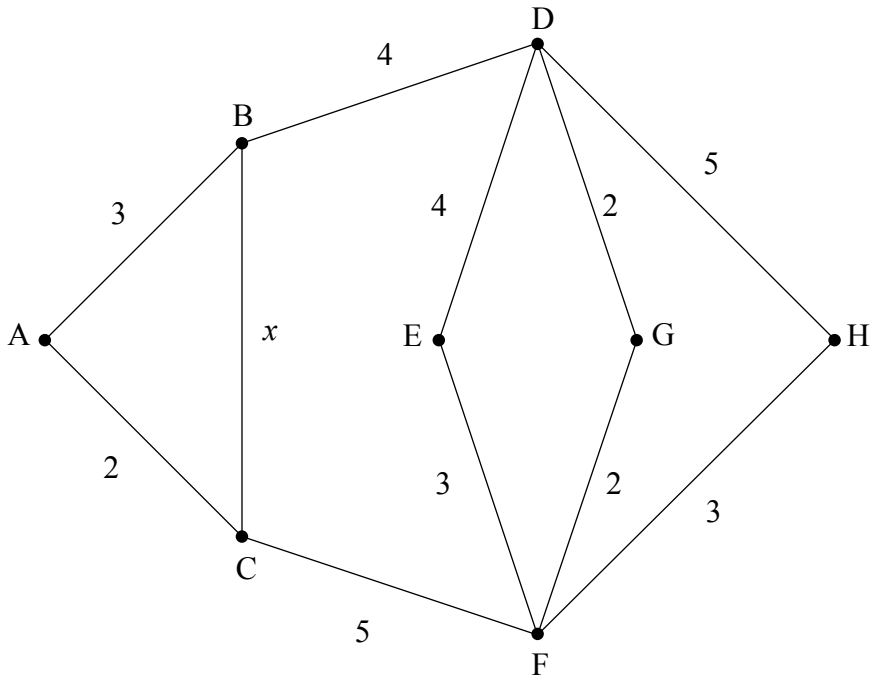
8. [Maximum mark: 5]

The weights on the following graph represent the lengths of different roads in kilometres.

diagram not to scale



(a) Write down the vertices with odd degree. [1]

The total length of the roads is $33 + x$ km.

(b) Find **two** expressions, in terms of x, for the shortest distance required to walk along all of the paths, beginning and ending at the same vertex.
Include in your answer the interval of values of x for which each expression is valid. [4]

(This question continues on the following page)

(Question 8 continued)

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9. [Maximum mark: 7]

(a) Find $\int \frac{8}{2x+3}\,\mathrm{d}x$. [3]

(b) Hence find the exact area between the curve $y = \frac{8}{2x+3}$, the x-axis and the lines $x = 0$ and $x = 6$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{N}$. [4]

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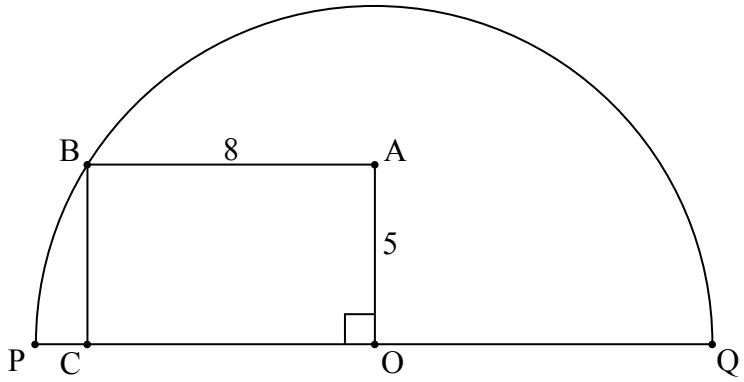
10. [Maximum mark: 5]

The following diagram shows a semicircle with centre O and diameter PQ. A rectangle $OABC$ is also shown, such that $AB = 8$ and $OA = 5$.

diagram not to scale



Find the length of the arc BQ.

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11. [Maximum mark: 8]

Let $\boldsymbol{R}(\alpha)$ be the matrix representing a rotation, counter-clockwise (anticlockwise) about the origin, through an angle of α.

(a) Write down $\boldsymbol{R}(2\alpha)$ as a 2×2 matrix. [2]

(b) Calculate $\boldsymbol{R}(\alpha) \times \boldsymbol{R}(\alpha)$. [2]

(c) Use your answers from part (a) and part (b) to

(i) explain why $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$.

(ii) show that $\cos(2\alpha) = 1 - 2\sin^2(\alpha)$. [4]

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12. [Maximum mark: 8]

Roma is told by the manufacturers of their phone that, when charging from 0%, the percentage P of charge can be modelled by $P = 100 - 100 \times 2^{-t}$, where t is the number of hours since the phone started charging.

(a) Find the time taken for the amount of charge to go from 0% to 40%, according to this model. [2]

Roma decides to test the model. They charge the phone from 0% and record the charge of the phone for different values of t. The results are shown in the following table.

t **hours**	1	2	3	4
P **percent**	48	74	86	91

(b) Find the sum of the square residuals (SS_{res}) when using $P = 100 - 100 \times 2^{-t}$ to fit this data. [4]

Roma now uses a quadratic function to model the charge in the phone. They calculate that, when using this model, the sum of the square residuals for these four points is 2.45.

(c) State one reason why Roma might prefer to use

(i) the quadratic model.

(ii) $P = 100 - 100 \times 2^{-t}$. [2]

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13. [Maximum mark: 6]

Let $z_1 = 4 + 5\mathrm{i}$.

(a) (i) Find $|z_1|$.

(ii) Find $\arg(z_1)$. [2]

Let $z_2 = 3\mathrm{e}^{2\mathrm{i}}$.

(b) Find the area of the triangle on an Argand diagram with vertices 0, z_1 and z_2. [4]

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14. [Maximum mark: 4]

Two model cars are driving along the same straight track. Car A leaves point O at $t = 0$ and its displacement (x_A metres) from O at t seconds is given by the equation

$$x_A = 4\sqrt{t}\text{, for } 0 \le t \le 9.$$

A faster car, B, leaves O at $t = 3$.

After starting its motion, the time taken by car B to reach any point on the track is half the time that was taken by A.

(a) Write down the equation for the displacement of B, x_B, in terms of t. [2]

(b) Find the value of t at which B catches up with A. [2]

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15. [Maximum mark: 6]

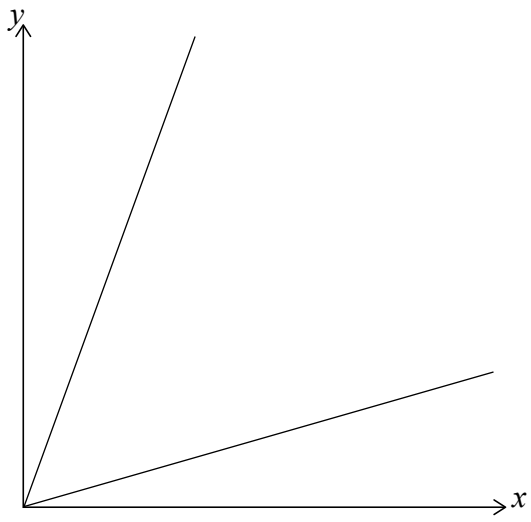
A system of differential equations of the form $\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by$, $\frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy$ has eigenvalues $\lambda = -1$ and $\lambda = 2$ with corresponding eigenvectors $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

The following incomplete phase portrait for this system, with $x, y \geq 0$, shows lines through $(0, 0)$ parallel to the eigenvectors.

(a) On the phase portrait

(i) show the direction of motion along the eigenvectors.

(ii) sketch one trajectory in each of the three regions. [3]



In the system described above, x and y are the population sizes of two species, X and Y. The population of Y is vulnerable, so it will be increased by adding more animals from a different area. Currently, $x = 252$ and $y = 60$.

(b) Find the minimum number of new animals from species Y that need to be added for the population not to reduce to 0 over time. [3]

(This question continues on the following page)

(Question 15 continued)

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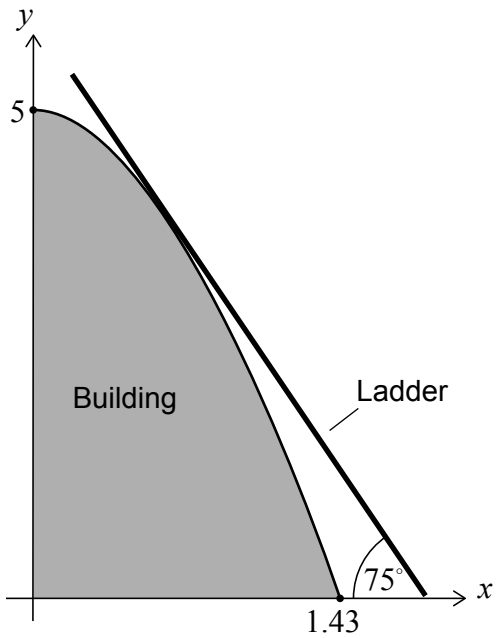
16. [Maximum mark: 8]

The cross section of the side of a building can be modelled by a curve with equation $y = 5\cos(1.1x), 0 \le x \le 1.43$, as shown in the following diagram. Distances are measured in metres.

diagram not to scale



A builder leans a straight ladder against the building to do repairs. For safety reasons, the angle between the ladder and the horizontal ground must be 75°.

Find the height above the ground at which the ladder touches the building.

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17. [Maximum mark: 9]

Phoebe opens a coffee shop, near to a well-established Apollo coffee shop.

After being open for a few months, Phoebe notices that

- 10% of customers who preferred the Apollo coffee shop in one month preferred her coffee shop the following month.
- 25% of customers who preferred her coffee shop in one month preferred the Apollo coffee shop the following month.

She decides to show these changes in the following transition matrix.

$$\begin{pmatrix} 0.9 & 0.25 \\ 0.1 & 0.75 \end{pmatrix}$$

The two eigenvalues for this matrix are 1 and 0.65. An eigenvector corresponding to the eigenvalue of 1 is $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

(a) Find an eigenvector corresponding to the eigenvalue of 0.65. [2]

A diagonal matrix of eigenvalues is $\boldsymbol{D} = \begin{pmatrix} 0.65 & 0 \\ 0 & 1 \end{pmatrix}$.

(b) Write down an expression for $\boldsymbol{D}^n$, giving your answer as a 2×2 matrix in terms of n. [1]

When Phoebe's coffee shop first opened, the Apollo shop had 7000 customers the previous month.

(c) Assuming all 7000 customers continue to go to one of these coffee shops, find an expression for the number that will favour Phoebe's coffee shop after n months. [6]

(This question continues on the following page)

(Question 17 continued)

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References:

7. Saddako, n.d. *Cheetah (Acinonyx jubatus) Running - stock photo.* [image online] Available at: https://www.gettyimages.co.uk/detail/photo/cheetah-running-royalty-free-image/523244194?phrase=cheetah+speed&adppopup=true [Accessed 2 May 2023]. Source adapted.

GlobalP, n.d. *Lion, Panthera leo, 8 years old, standing - stock photo.* [image online] Available at: https://www.gettyimages.co.uk/detail/photo/lion-panthera-leo-8-years-old-standing-royalty-free-image/134976936?phrase=Lion+standing&adppopup=true [Accessed 2 May 2023]. Source adapted.

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